## Direct Proportionality

$$
y=3 x
$$



As $x$ increases, $y$ increases proportionally.
$Y$ is directly proportional to $x$ because when $x=0, y=0$.

| $x$ | $3 x$ | $y$ |
| :--- | :--- | :--- |
| 1 | 3 | 3 |
| 2 | 6 | 6 |
| 3 | 9 | 9 |
| 4 | 12 | 12 |

1) IF you double $X$, ( $X=1$ to $X=2$ ) by what factor does $Y$ change? by 2 since $Y$ changed from 3 to 6

$$
\text { Multiplicative Change Factors: X by__2_ Y by ___ } 2
$$

2) If you triple $X$, ( $X=1$ to $X=3)$ by what factor does $Y$ change?

Change Factors: X by
Y by $\qquad$
3) If you quadruple $X$, ( $X=1$ to $X=4)$ by what factor does $Y$ change?

Change Factors: X by _ Y by $\qquad$
4) For this Data, is $Y$ proportional to $X$ ? If so, by what factor? $\qquad$
This factor is called the constant of proportionality.
Hint: If you graph $\qquad$ Y $\qquad$ vs $\qquad$ X $\qquad$ , how can you obtain the proportionality constant
mathematically? Compute the $\qquad$ of the line whose formula is $\Delta Y / \Delta X$.

KEY IDEA: 1,2, and 3 illustrate linear proportionality, as you change $X, Y$ changes the same way. Do you realize that the "linear" equation is $Y=3 X$ where $Y$ is on the vertical axis and $X$ is on the horizontal axis? That's a straight line and straight lines are easy to analyze!

## Squared Proportionality



| $X$ | $Y=3 x^{2}$ | $X^{2}$ | $Y=3 X^{2}$ |
| :--- | :--- | :--- | :--- | :--- |
| 1 | 3 | 1 | 3 |
| 2 | 12 | 4 | 12 |
| 3 | 27 | 9 | 27 |
| 4 | 48 | 16 | 48 |
| 5 | 75 | 25 | 75 |
| 6 | 108 | 36 | 108 |
| 7 | 147 | 49 | 147 |
|  |  |  |  |

1) IF you double $X$, ( $X=1$ to $X=2$ ), by what factor does $Y$ change?

Multiplicative Change Factors: X by $\qquad$ Y by $\qquad$
2) If you triple $X,(X=1$ to $X=3)$, by what factor does $Y$ change?

Change Factors: X by $\qquad$ Y by $\qquad$
3) If you increase $X^{2} 4$ times $\left(X^{2}=1\right.$ to $\left.X^{2}=4\right)$ by what factor does $Y$ change?

Change Factors: $X^{2}$ by $\qquad$ Y by $\qquad$
4) If you increase $X^{2} 9 \operatorname{times}\left(X^{2}=1\right.$ to $\left.X^{2}=9\right)$ by what factor does $Y$ change?

Change Factors: $X^{2}$ by $\qquad$ Y by $\qquad$
5) Which question set has the simpler pattern? Between questions 1 \& 2 or between questions 3 \& 4?
6) Based on the pattern, is $Y$ proportional to $X$ or is $Y$ proportional to $X^{2}$ ? Why?
7) Using your choice from question (6), what is the value of the proportionality constant?

Hint: If you graph $\qquad$ vs $\qquad$ , how can you obtain the constant mathematically?

KEY IDEA: Do you realize that the "linear" equation is $Y=3 X^{2}$ where $Y$ is on the vertical axis and $X^{2}$ is on the horizontal axis. That's how one obtains a straight line from a curvy line. Beautiful!

Inverse Proportionality

## Fill Time(hr)

As Hose Area increases, Fill time decreases.

## Side Opening Parabola

| Filling the Swimming Pool |  |  |  |
| :--- | :--- | :--- | :--- |
| Hose Area <br> Opening(cm) | Fill Time <br> (hours) | $1 /$ Hose Area <br> $(\mathrm{cm})^{-2}$ | Fill Time <br> (hours) |
| 3 | 72 | 0.3333 | 72 |
| 18 | 12 | 0.0556 | 12 |
| 33 | 7 | 0.0303 | 7 |
| 48 | 3.5 | 0.0159 | 3.5 |
| 63 | 2 | 0.0093 | 2 |
| 108 |  |  |  |

1) As the Hose Area increases from $\mathbf{3} \mathbf{c m}^{2}$ to $\mathbf{1 8} \mathbf{c m}^{2}$, by what factor does the Fill Time change?

Multiplicative Change Factors: Hose Area by
$\qquad$ 6 $\qquad$ Fill Time by $\qquad$ 1/6 $\qquad$
2) As the Hose Area increases from $\mathbf{3} \mathbf{c m}^{\mathbf{2}}$ to $\mathbf{4 8} \mathbf{c m}^{\mathbf{2}}$, by what factor does the Fill Time change?

Change Factors: Hose Area by $\qquad$ Fill Time by $\qquad$
3) As the $1 /$ Hose Area decreases from $0.3333 \mathrm{~cm}^{-2}$ to $0.0556 \mathrm{~cm}^{-2}$, by what factor does the Fill Time change?

Change Factors: 1/Hose Area by $\qquad$ Fill Time by $\qquad$
4) As the $1 /$ Hose Area decreases from $0.3333 \mathrm{~cm}^{-2}$ to $0.0208 \mathrm{~cm}^{-2}$, by what factor does the Fill Time change?

Change Factors: 1/Hose Area by $\qquad$ Fill Time by $\qquad$
5) Which is the simpler pattern, between questions $1 \& 2$ or between questions $\mathbf{3} \& 4$ ?
6) Is Fill Time proportional to Hose Area or is Fill Time proportional to $1 /$ Hose Area?
7) If you graph $\qquad$ vs $\qquad$ , how can you obtain the constant mathematically?
8) Optional: Based on your answer to (6), what is the value of the constant of proportionality?

Optional: Write the "linear" equation for this relationship between Fill Time \& Hose Area:

Pendulum Period(T) vs Length(L)


| $\mathrm{L}(\mathrm{cm})$ | $\mathrm{T}(\mathrm{s})$ | $\mathrm{L}(\mathrm{cm})$ | $\mathrm{T}^{2}\left(\mathrm{~s}^{2}\right)$ |
| :--- | :--- | :--- | :--- |
| 1 | 2 | 1 | 4 |
| 5 | 4.47 | 5 | 19.98 |
| 10 | 6.32 | 10 | 39.94 |
| 20 | 8.90 | 20 | 79.21 |
| 40 | 12.6 | 40 | 158.8 |
| 80 | 17.9 | 80 | 320.4 |
| 120 | 21.9 | 120 | 480 |

1) As $L$ changes by a factor of 5 from 1 cm to 5 cm, by what factor does T change?

Multiplicative Change Factors: L by $\qquad$ T by $\qquad$
2)) As $L$ changes by a factor of 5 from 1 cm to 5 cm , by what factor does $T^{2}$ change?

Change Factors: L by $\qquad$ $\mathrm{T}^{2}$ by $\qquad$
3) As $L$ changes by a factor of 20 from 1 cm to 20 cm , by what factor does T change?

Change Factors: L by $\qquad$ T by $\qquad$
4) As $L$ changes by a factor of 20 from 1 cm to 20 cm , by what factor does $\mathrm{T}^{2}$ change?

## Change Factors: L by

$\qquad$ $\mathrm{T}^{2}$ by $\qquad$
5) Based on your answers to the questions above, is $T$ proportional to $L$ or is $T^{2}$ proportional to L? Explain Why?
6) Based on your answer to (5), what is the value of the constant of proportionality?
7) If you graph $\qquad$ vs $\qquad$ , how can you obtain the constant mathematically?

Optional: Write the "linear" equation for this relationship between T \& L: $\qquad$

